Towards an operator for merging taxonomies

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Taaable (http://taaable.fr)
WikiTaaable (http://wikitaaable.loria.fr)
Kolflow project (http://kolflow.univ-nantes.fr)

Outline of the talk

- Context and motivation
- Merging taxonomies
- Conclusion and future work

Context and motivation

Taaable and WikiTaaable

`http://taaable.fr`

`http://wikitaaable.loria.fr`

- Taaable: a CBR system that reuses a cooking recipe base
- WikiTaaable: a semantic wiki for the Taaable knowledge base including a taxonomical domain ontology

DSMW

- MW = MediaWiki, a wiki engine
- SMW = Semantic MW, a semantic wiki engine
- DSMW = Distributed SMW
  - Several WikiTaaables

Kolflow http://kolflow.univ-nantes.fr

- Man-machine collaboration in continuous knowledge construction flows

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- Man-machine collaboration in continuous knowledge construction flows
- Merging the contents of two semantic wikis
  - The textual parts
  - The knowledge parts
- Often, the two semantic wikis come from another one, so they are quite similar

Knowledge representation in a semantic wiki: mainly class-superclass relations

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$$\texttt{Melon} \sqsubseteq \texttt{Fruit}$$

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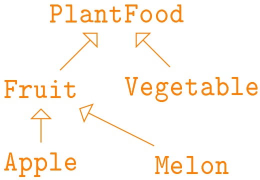
$$\texttt{Melon} \sqsubseteq \texttt{Fruit}$$

$$\forall x \quad \texttt{Melon}(x) \Rightarrow \texttt{Fruit}(x)$$

Merging taxonomies

Taxonomy language

- $\mathcal{L}_{\mathcal{T}}$: language of taxonomies
  A formula of $\mathcal{L}_{\mathcal{T}}$: $A \sqsubseteq B$
  Deductive inferences based on the transitivity of $\sqsubseteq$



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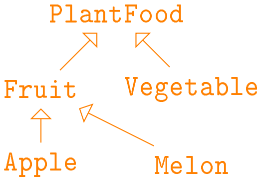

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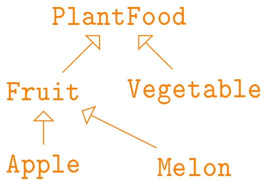
  Deductive inferences based on the transitivity of $\sqsubseteq$
- A taxonomy ψ: a finite set of formulas of $\mathcal{L}_{\mathcal{T}}$
- Example:

$$\psi = \left\{ \begin{array}{ll} \texttt{Apple} \sqsubseteq \texttt{Fruit}, & \texttt{Melon} \sqsubseteq \texttt{Fruit}, \\ \texttt{Fruit} \sqsubseteq \texttt{PlantFood}, & \texttt{Vegetable} \sqsubseteq \texttt{PlantFood} \end{array} \right\}$$

$$\mathcal{V}(\psi) = \{\texttt{Apple}, \texttt{Fruit}, \texttt{Melon}, \texttt{PlantFood}, \texttt{Vegetable}\}$$



Merging two taxonomies, what does it mean?

- Usual intuition of merging ψ_1 and ψ_2:
  minimally modify ψ_1 and ψ_2 into ψ_1' and ψ_2' so that their *conjunction* is consistent

$$\Delta(\{\psi_1, \psi_2\}) = \psi_1' \wedge \psi_2'$$

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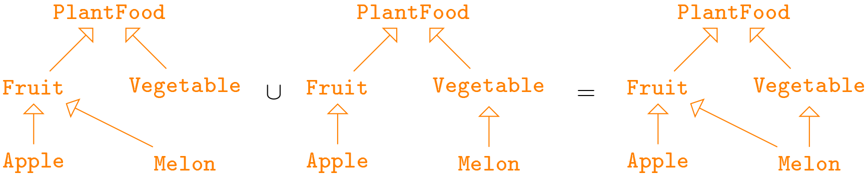

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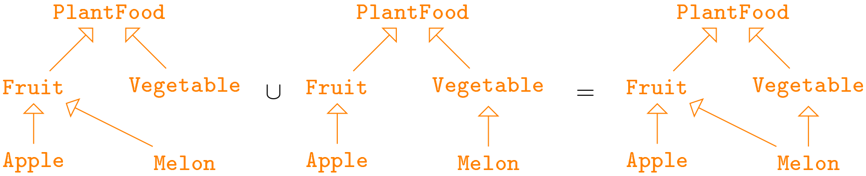

- Another definition of $\wedge$ is proposed for taxonomies.

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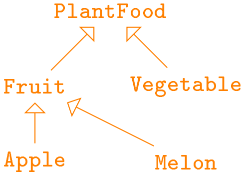
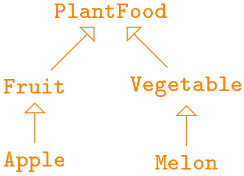
$\psi_1 =$ 

$\psi_2 =$ 

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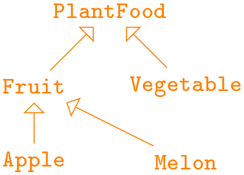

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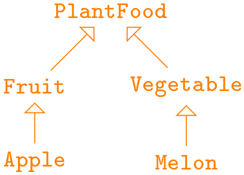

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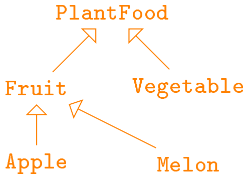
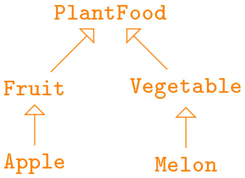
$\psi_1 =$ 

$\psi_2 =$ 

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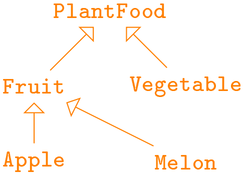
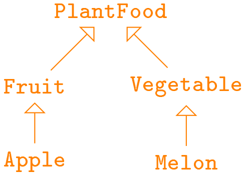
$\psi_1 =$ 

$\psi_2 =$ 

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  2. $\psi_2 \models \texttt{Melon} \not\sqsubseteq \texttt{Fruit}$ (closed-world assumption, CWA)

$$\frac{\psi \not\models A \sqsubseteq B}{A \not\sqsubseteq B}\,\textsf{CWA}$$

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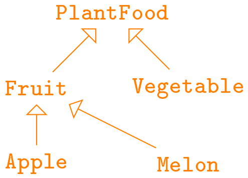

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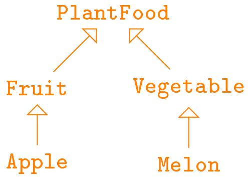

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- A taxonomy ψ of $\mathcal{L}_{\mathcal{T}}$ considered under CWA:

$$\begin{aligned}\widehat{\psi} &= \{A \sqsubseteq B \mid A, B \in \mathcal{V}(\psi), \quad \psi \models A \sqsubseteq B\} \\ &\cup \{A \not\sqsubseteq B \mid A, B \in \mathcal{V}(\psi), \quad \psi \not\models A \sqsubseteq B\}\end{aligned}$$

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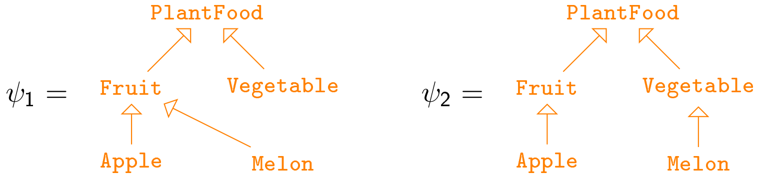

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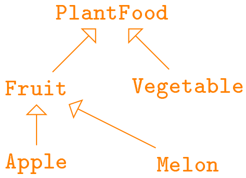
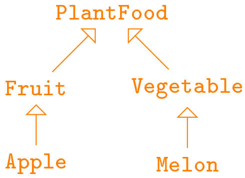
$\psi_1 =$ 

$\psi_2 =$ 

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- In the example, $\psi_1 \wedge \psi_2$ is inconsistent, since $\psi_1 \wedge \psi_2 \supseteq \{\texttt{Melon} \sqsubseteq \texttt{Fruit}, \texttt{Melon} \not\sqsubseteq \texttt{Fruit}\}$

Taxonomy language with negations: $\mathcal{L}_{\mathcal{T}}^{\neg}$

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- $\widehat{\cdot} : \psi \in \mathcal{L}_\mathcal{T} \mapsto \widehat{\psi} \in \mathcal{L}_\mathcal{T}^\neg$
- $\mathcal{L}_\mathcal{T}^\neg$'s formulas: $A \sqsubseteq B$ and $A \not\sqsubseteq B$

$$
\begin{array}{lll}
A \sqsubseteq B & \text{means} & \forall x \quad A(x) \Rightarrow B(x) \\
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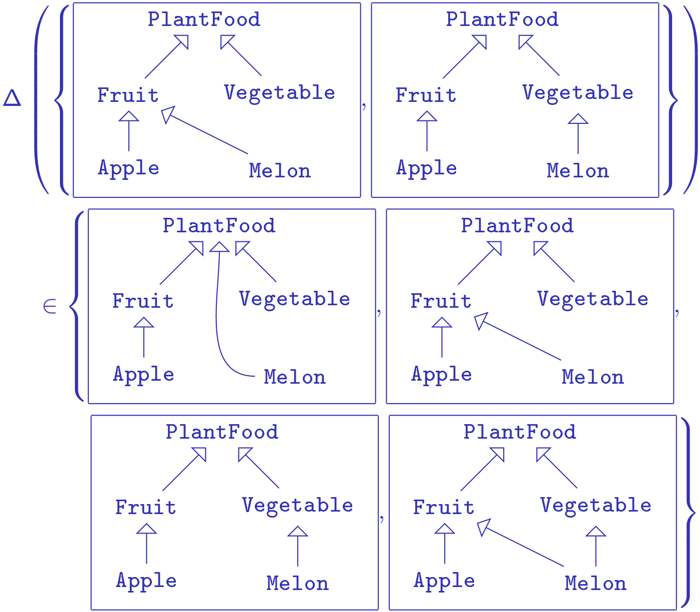
- Remark: $\mathcal{L}_{\mathcal{T}}^{\neg}$ is not propositionnaly closed

Expected result of merging, on the example



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7. **return** deductive reduction of Γ

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- Complexity (of a straightforward algorithm): polynomial in $|\alpha|$ + exponential in $|\delta|$

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- Integrating the user in the choice process (and reusing previous choices of users)
- Implementation, test, optimisation